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## RAINDROP-SIZE DISTRIBUTIONS IN A HILLY REGION

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SUMMARY

Raindrop-size distributions have been measured in a hilly region of North Wales for the summers of 1972–74 and used to calculate empirical relationships between certain parameters important in radar and radio meteorology.

The observed rainfall rates were between 0·1 mm $h^{-1}$ and 47·9 mm $h^{-1}$, and about 34 000 raindrop-size distributions were used to calculate various relationships by a linear regression technique.

The drop-size distributions at the lower rainfall rates showed close agreement with the Marshall–Palmer exponential distribution, but for higher rates there was an increasingly noticeable deficiency of both very small and very large drops.

The radar reflectivity factor ($Z$) and rate of rainfall ($R$) were calculated from each distribution, and the empirical relationship of the form $Z = AR^B$, where $A$ and $B$ are 'constants'. The constants calculated were higher than those measured by most other authors who used a similar regression technique. A comparison of areal radar measurements using the widely accepted value for the exponent (constant $B$) of 1·6 and the value of 1·8 derived from distrometer data from North Wales produced an improvement in the accuracy of measurements of about 10 per cent for rainfall rates less than 1 mm $h^{-1}$, and 2–3 per cent for rainfall rates equal to or greater than 1 mm $h^{-1}$.

The $Z$–$R$ relationships have been calculated for various synoptic types and storm by storm. These results show wide variability and it is concluded that radar systems should be calibrated using a few telemetering gauges in the area of interest.

Empirical relationships between rate of rainfall, attenuation, median drop-size diameter and liquid water content have also been derived, and these confirm the results of other authors.

### INTRODUCTION

Size distributions of raindrops have been measured and interpreted in many parts of the world, largely because they are important in determining the accuracy of measurements of precipitation using radar and assessing the problem of attenuation in radio communication caused by hydrometeors.

It has been shown that the drop-size distributions are highly variable in space and time, and because of this the empirical relationships between the various parameters that can be derived from them (such as radar reflectivity factor, rate of rainfall and liquid water content) are subject to wide variations. (For a survey of these relationships see for example Stout and Mueller (1968).)

As a part of the Dee Weather Radar Project (Harrold *et alii* (1974)), raindrop spectra have been measured at two sites in North Wales to investigate their variability in a hilly region where accurate measurements of areal precipitation are difficult to achieve. Harrold *et alii* showed that although radar measurements of precipitation could be made with sufficient accuracy for many hydrological purposes, one of the major limitations to the accuracy obtainable was the change in the drop-size distribution over the area of measurement. The purpose of this paper is to describe the results derived from the very large number of drop-size spectra measured, to show the difficulties in achieving accurate rainfall measurements from a radar system, and to calculate the attenuation of microwave radiation caused by rainfall.

## Instrumentation

The raindrop spectra were measured using a Joss distrometer* (Joss and Waldvogel (1967)). This instrument has a collecting area of 50 $cm^2$ and a crystal clock provides time pulses at one-minute intervals. The mechanical momentum of each raindrop is transformed into an electric pulse, and thereby the size of each drop is measured using a pulse-height analyser. Each drop is classified into one of 20 size categories ranging from 0·3 mm to 6·0 mm in diameter. However, it was found that in the first eight size intervals the numbers of drops measured alternated between many and a few for consecutive sizes, a defect attributed to slight inaccuracy in calibration. This was overcome by pairing size intervals, therefore reducing the first eight to four larger size ranges.

The two sites chosen were at Carrog and Bala (see Figure I). The instrument was at Carrog during the summers of 1972 and 1974, and at Bala in the summer of 1973. The distrometer was mounted with its rim flush with the surrounding ground and close to a Plessey MM37 tipping-bucket rain-gauge which served as a check by providing an independent measure of rainfall. A comparison of gauge- and distrometer-measured rainfall is shown in Figure 2; the annual totals for the coincident periods are shown in Table I.

TABLE I—RAINFALL MEASURED BY GAUGE AND DISTROMETER

| Year | Gauge-measured rainfall | Distrometer-measured rainfall | Hours of data |
|---|---|---|---|
| | *mm* | *mm* | |
| 1972 | 104·0 | 119·1 | 1580 |
| 1973 | 145·6 | 154·6 | 2256 |
| 1974 | 93·6 | 82·2 | 1275 |

Aniol (1974) indicated that a distrometer underestimates the rainfall in strong winds, an occurrence which would be expected in North Wales. Table I does not substantiate this but both Figure 2 and Table I demonstrate that, though errors are likely both with gauge and distrometer, the measurements are sufficiently alike for us to assume that the distrometer correctly measures raindrop distributions.

* Dr Joss informs us that the word *distrometer* is derived from *distr*ibution *meter* and is the correct spelling for the instrument developed by him and Dr Waldvogel. The spelling *disdrometer* had previously been used for another instrument (Editor).

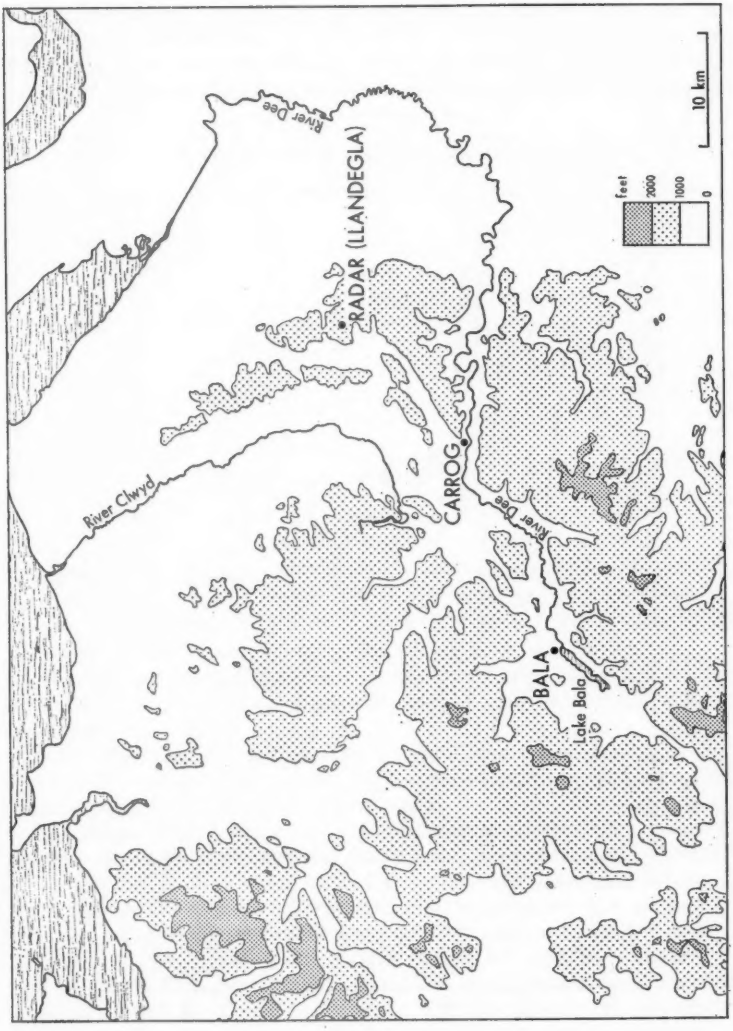


FIGURE 1—A MAP OF NORTH WALES SHOWING THE LOCATIONS OF THE DISTROMETER SITES AT BALA AND CARROG

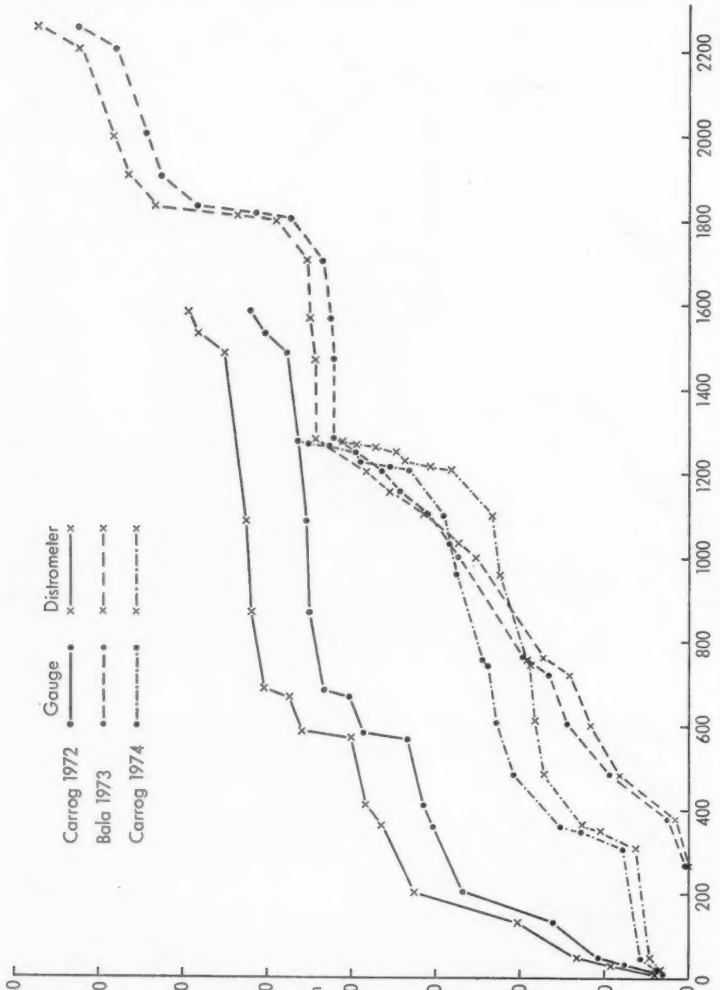


FIGURE 2—CUMULATIVE RAINFALL MEASURED BY GAUGE AND DISTROMETER

## Drop-size distributions

The drop-size distribution can be defined by the summation over all sizes of the number of drops in a unit volume ($N_D$) with diameters in the range $D$ to $D+\Delta D$.

Although the distrometer measures the number of drops ($N$) of given sizes hitting a sensor of area 50 cm$^2$ per minute (i.e. a flux) it is normal to express the drop-size distribution in terms of $N_D$ which is the number of drops per unit volume per unit size interval with a diameter $D$. The size intervals used and their respective raindrop terminal velocities are shown in Table II.

TABLE II—TERMINAL VELOCITY OF RAINDROPS

| Category | Minimum diameter *mm* | Maximum diameter *mm* | Terminal speed $m\,s^{-1}$ |
|---|---|---|---|
| 1 | 0·3 | 0·5 | 1·62 |
| 2 | 0·5 | 0·7 | 2·47 |
| 3 | 0·7 | 1·0 | 3·36 |
| 4 | 1·0 | 1·4 | 4·61 |
| 5 | 1·4 | 1·6 | 5·41 |
| 6 | 1·6 | 1·8 | 5·87 |
| 7 | 1·8 | 2·1 | 6·40 |
| 8 | 2·1 | 2·4 | 6·98 |
| 9 | 2·4 | 2·7 | 7·48 |
| 10 | 2·7 | 3·0 | 7·88 |
| 11 | 3·0 | 3·3 | 8·21 |
| 12 | 3·3 | 3·7 | 8·50 |
| 13 | 3·7 | 4·1 | 8·77 |
| 14 | 4·1 | 4·5 | 8·95 |
| 15 | 4·5 | 5·0 | 9·05 |
| 16 | 5·0 | 6·0 | 9·12 |

Marshall and Palmer (1948) related their drop-size distributions measured in Canada by $N_D=N_0e^{-\lambda D}$ where $N_0$ is the value of $N_D$ for $D=0$ and $\lambda$ is a function of the rate of rainfall, $\lambda=41R^{-0\cdot21}$ cm$^{-1}$. The results obtained in North Wales are compared with this widely accepted distribution.

The calculated parameters have been compared using the empirical relationship in the form of the afore-mentioned $Z$–$R$ relationship i.e. $Z=AR^B$ where $A$ and $B$ are 'constants'.

Stout and Mueller (1968), in their review, have shown that these 'constants' have in fact widely varying values dependent on location, synoptic weather type and probably the method of measurement.

The mean raindrop-size distributions for a number of selected rates of rainfall have been computed for each of the three summers, by averaging the drop spectra for all minutes when the rainfall rate was within specific bounds. For example, all minutes with rainfall rates between 0·6 and 1·5 mm h$^{-1}$ have been averaged to give the mean distribution for about 1 mm h$^{-1}$ (0·95 mm h$^{-1}$ in fact). If only minutes with exactly 1 mm h$^{-1}$ had been chosen, the data sample would have been rather small. The mean distributions for three fixed rates of rainfall are shown in Figure 3. For comparison the Marshall–Palmer (1948) distributions for the equivalent rainfall rates are also included.

It can be seen that the mean distributions for each year were very similar to one another and to the appropriate Marshall–Palmer distribution. However, there was a tendency for fewer small and fewer very large drops to be measured than would have been expected from the Marshall–Palmer distribution. These

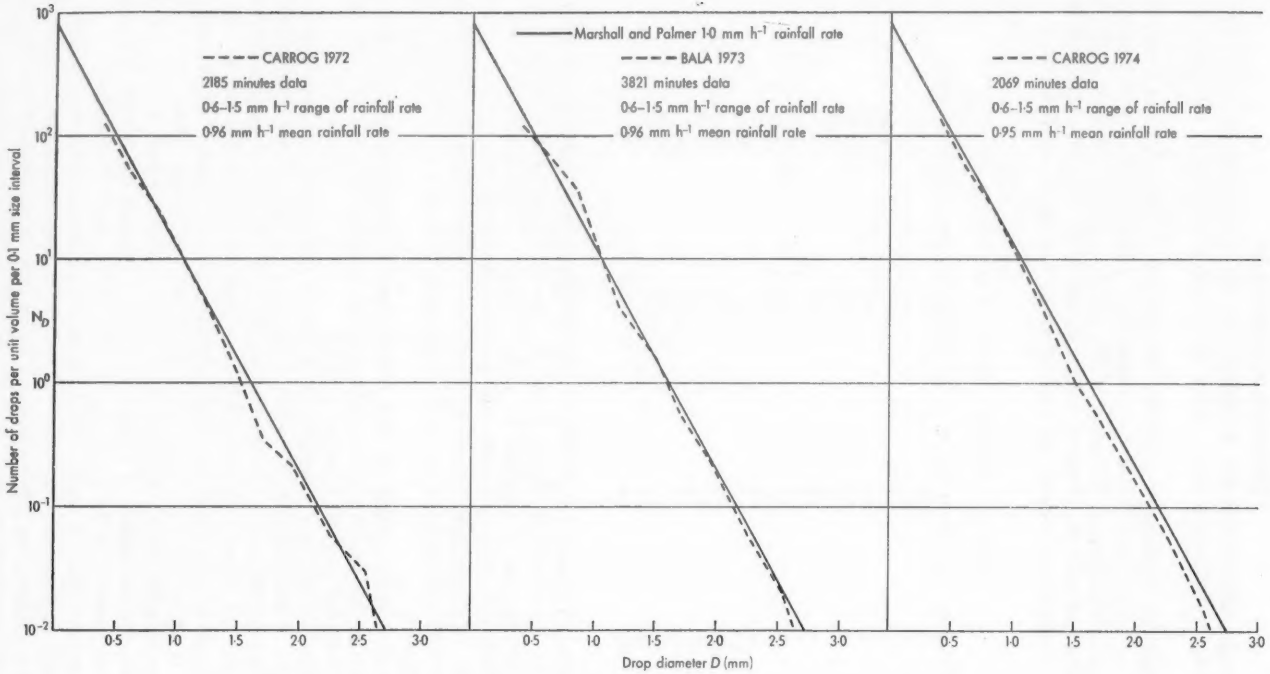


FIGURE 3—THE MEAN DROP SIZE DISTRIBUTIONS FOR THREE SELECTED RATES OF RAINFALL

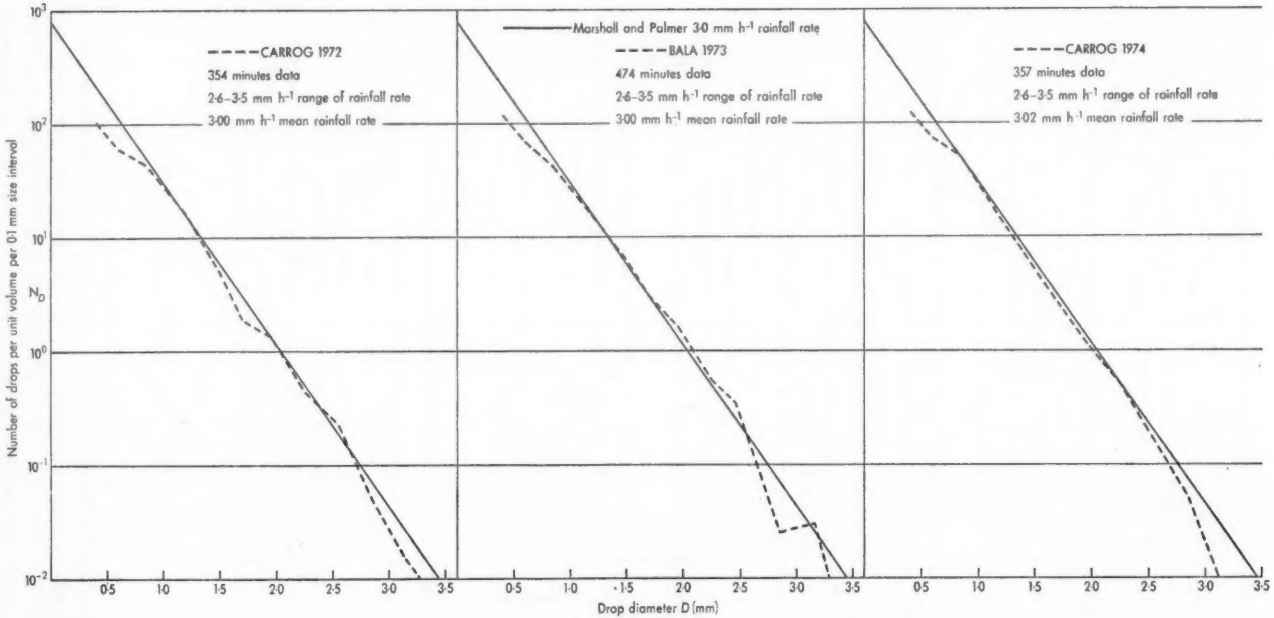
Marshall and Palmer 3·0 mm h⁻¹ rainfall rate
CARROG 1972
354 minutes data
2·6–3·5 mm h⁻¹ range of rainfall rate
3·00 mm h⁻¹ mean rainfall rate
BALA 1973
474 minutes data
2·6–3·5 mm h⁻¹ range of rainfall rate
3·00 mm h⁻¹ mean rainfall rate
CARROG 1974
357 minutes data
2·6–3·5 mm h⁻¹ range of rainfall rate
3·02 mm h⁻¹ mean rainfall rate
Number of drops per unit volume per 0·1 mm size interval
$N_D$
Drop diameter D (mm)

FIGURE 3—*continued*

**Carrog 1972 mean rainfall rate should read 2.98 mm h⁻¹**

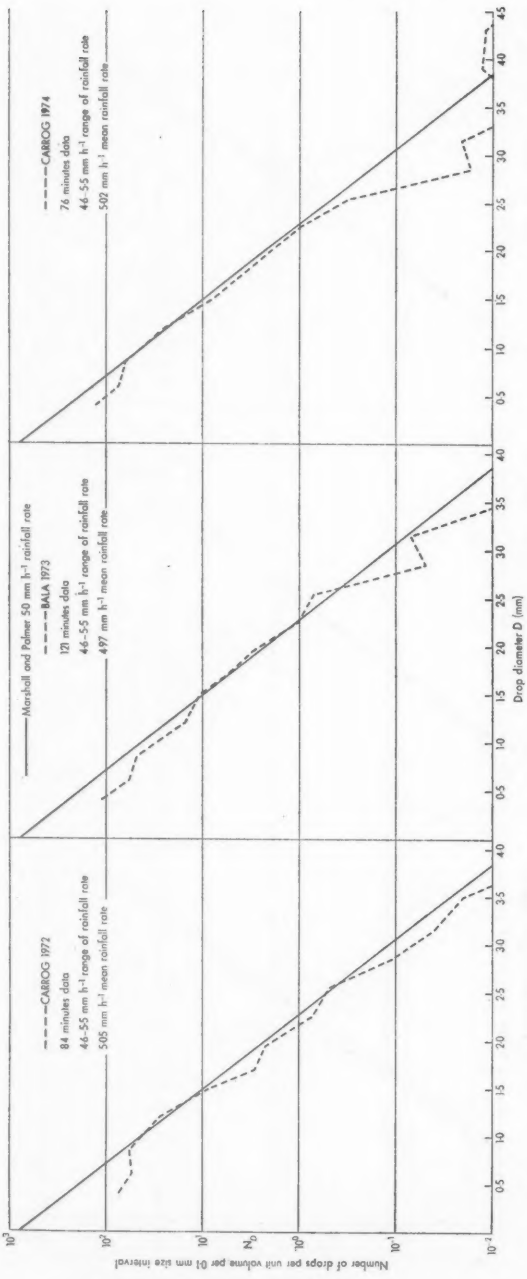
Marshall and Palmer 50 mm h⁻¹ rainfall rate
----CARROG 1972
84 minutes data
46–55 mm h⁻¹ range of rainfall rate
505 mm h⁻¹ mean rainfall rate
----BALA 1973
121 minutes data
46–55 mm h⁻¹ range of rainfall rate
497 mm h⁻¹ mean rainfall rate
----CARROG 1974
76 minutes data
46–55 mm h⁻¹ range of rainfall rate
502 mm h⁻¹ mean rainfall rate
Number of drops per unit volume per 0·1 mm size interval
$N_D$
Drop diameter D (mm)

**FIGURE 3**—*continued*

measured deficiences become more marked at higher rainfall rates, as demonstrated in Figure 4, which shows the mean raindrop spectra for a number of rates from 0·95 mm $h^{-1}$ to 38·3 mm $h^{-1}$ at Carrog in 1972. (At the higher rates shown in Figure 4, data covering a very few minutes were included in the mean distributions.) The Marshall–Palmer distributions for 10 mm $h^{-1}$ and 40 mm $h^{-1}$ are also shown.

It is worth noting that the maximum numbers of drops at higher rainfall rates occur at about 1·2 mm diameter. Peaks in the drop-size distributions have been observed by Caton (1966) using Doppler radar in England as the measuring instrument and by Hardy (1963) in the United States using a photoelectric spectrometer, although their maxima were for drop diameters between 0·5 and 1·0 mm.

Some authors (e.g. Joss and Waldvogel (1970)) have subdivided their data into synoptic types and found different distributions for each. The data from North Wales have been treated in this manner, by selecting the periods of rainfall ahead of warm fronts, cold fronts and occlusions, rain associated with depressions (called cyclonic) and shower rain (including thunderstorms) and then finding the mean drop-size distributions for given rainfall rates. It was found that for given rain types and for fixed rates the mean drop spectra were very similar in the three years. An example is shown in Figure 5 where the drop-size distributions for rainfall rates of about 3 mm $h^{-1}$ ahead of cold fronts are plotted for the three years, together with the Marshall–Palmer distribution for 3 mm $h^{-1}$. Once again the shortage of very small and very large drops can be seen.

Figure 6 shows the mean distributions for four weather types (occlusions did not provide sufficient data) at about 1 mm $h^{-1}$ for Carrog 1972. It can be seen that for drops larger than 2 mm diameter, the four distributions are very similar. Below 2 mm there is a considerable difference between the types, in particular between frontal rain and showers. Surprisingly showers have more than twice as many small drops as warm frontal rain but only about 65 per cent as many medium-size drops.

## The relationship between rainfall intensity and radar reflectivity

The parameters derived most frequently from drop-size distributions are the rate of rainfall $R$ (in mm $h^{-1}$) and the radar reflectivity factor $Z$ (in $mm^6m^{-3}$). It is widely accepted that an empirical relationship between them can be used to estimate rates of precipitation from radar measurements. However, the results of other authors around the world show that the relationship is highly variable and the use of a fixed equation could lead to serious error in the radar estimate of rainfall intensity. $N_D$ and $D$ are known for each drop-size distribution and since

$$Z \propto \sum_{i=1}^{16} N_{D,1} D^6 \text{ and } R \propto \sum_{i=1}^{16} N_{D,1} D^3,$$

values of $Z$ and $R$ maybe calculated for each distribution.

The relationship between $Z$ and $R$ is usually expressed in the form $Z = AR^B$. However, since $R$ is here regarded as the dependent variable, the least-squares fit linear regression is of log $R$ on log $Z$.

This procedure was used in the results which follow, the drop-size distributions being measured by the distrometer over 1-minute periods. Table III shows

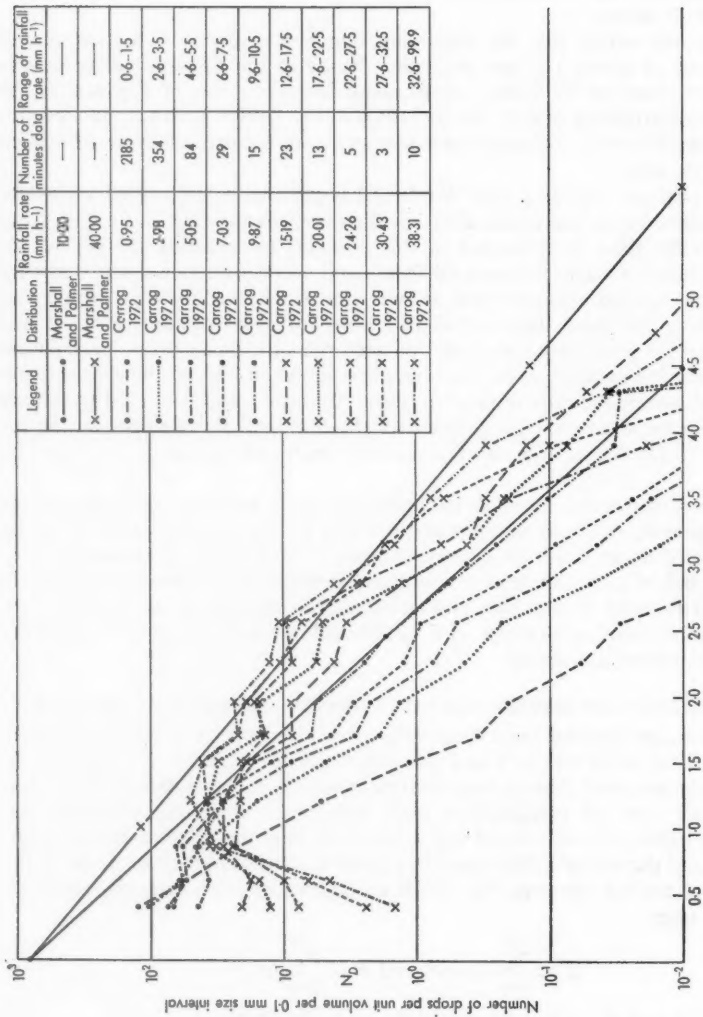


FIGURE 4—THE MEAN DROP SIZE DISTRIBUTIONS FOR A VARIETY OF RATES OF RAINFALL MEASURED AT CARROG IN 1972

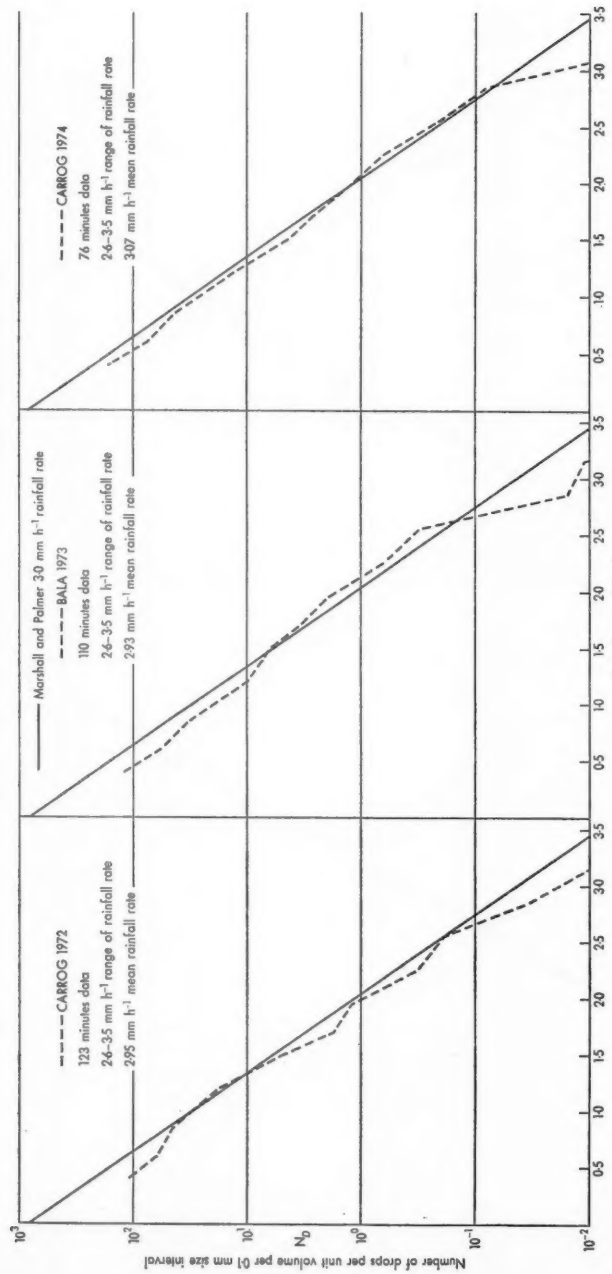
Marshall and Palmer 30 mm h⁻¹ rainfall rate
BALA 1973
110 minutes data
24–35 mm h⁻¹ range of rainfall rate
293 mm h⁻¹ mean rainfall rate
CARROG 1974
76 minutes data
24–35 mm h⁻¹ range of rainfall rate
307 mm h⁻¹ mean rainfall rate
CARROG 1972
123 minutes data
24–35 mm h⁻¹ range of rainfall rate
295 mm h⁻¹ mean rainfall rate
Number of drops per unit volume per 01 mm size interval
N_D
Drop diameter D (mm)

FIGURE 5—THE MEAN DROP SIZE DISTRIBUTIONS FOR RAIN AHEAD OF COLD FRONTS WITH AN INTENSITY OF ABOUT 3 mm $h^{-1}$

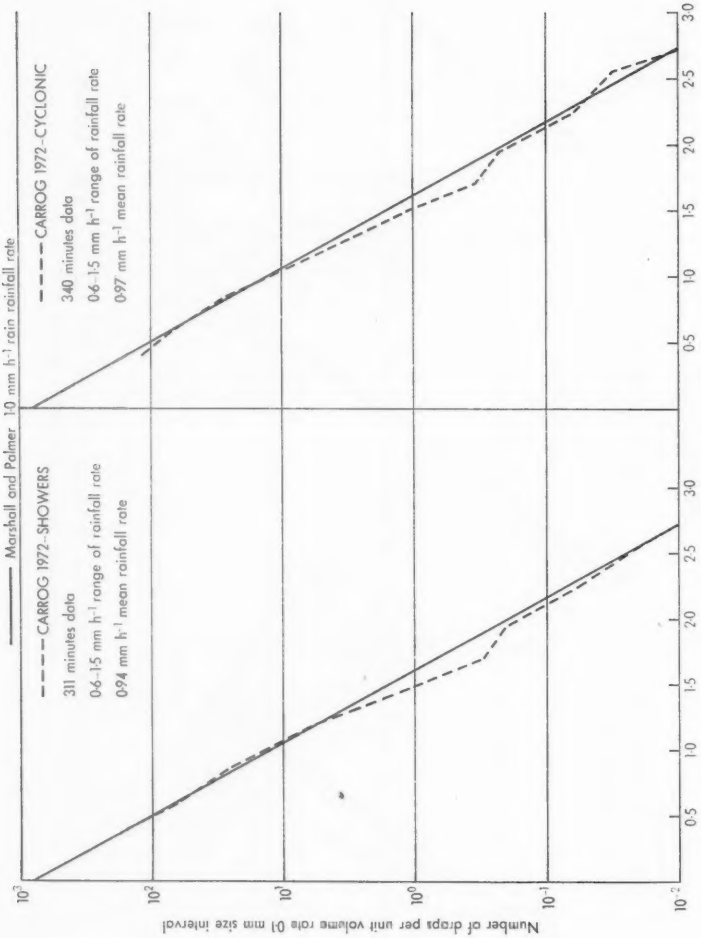
Marshall and Palmer 1·0 mm h⁻¹ rain rainfall rate
CARROG 1972–SHOWERS
311 minutes data
0·6–1·5 mm h⁻¹ range of rainfall rate
0·94 mm h⁻¹ mean rainfall rate
CARROG 1972–CYCLONIC
340 minutes data
0·6–1·5 mm h⁻¹ range of rainfall rate
0·97 mm h⁻¹ mean rainfall rate
Number of drops per unit volume m⁻³ 0·1 mm size interval
Drop diameter D (mm)

**FIGURE 6—THE MEAN DROP SIZE DISTRIBUTIONS FOR VARIOUS RAINFALL TYPES WITH AN INTENSITY OF ABOUT 1 mm $h^{-1}$ MEASURED AT CARROG IN 1972**

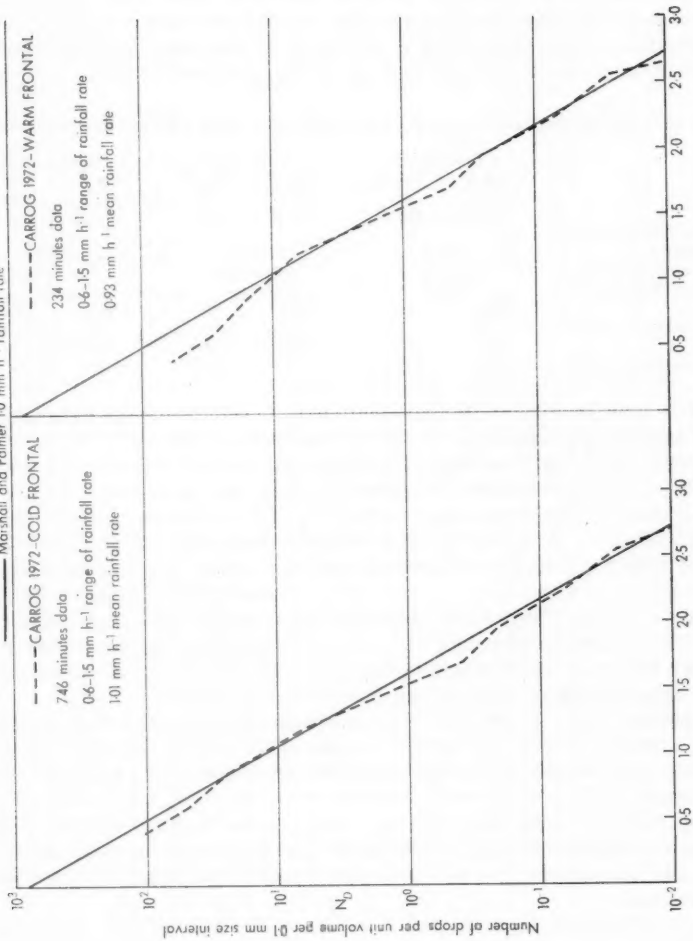


**FIGURE 6—*continued***

the $Z$–$R$ relationships in the three separate years calculated from log $R$ on log $Z$ linear regression for each minute when a rainfall rate of 0·1 mm $h^{-1}$ or more was measured. The Table also shows the number of minutes of data (that is, the number of drop-size spectra used in each year's regression), the correlation coefficient, the range of measured rates of rainfall and the standard deviation of the constants $A$ and $B$.

$Z$–$R$ relationships for different parts of the world have been quoted by several authors, but although a least-squares method was used it is not always stated whether a regression of log $Z$ on log $R$ or vice versa was calculated. Table III includes results by both methods for comparison.

TABLE III—RADAR REFLECTIVITY ($Z$) AND RAINFALL RATE ($R$) RELATIONSHIPS

| | Carrog 1972 | | Bala 1973 | | Carrog 1974 | |
|---|---|---|---|---|---|---|
| | log $R$ on log $Z$ | log $Z$ on log $R$ | log $R$ on log $Z$ | log $Z$ on log $R$ | log $R$ on log $Z$ | log $Z$ on log $R$ |
| Number of raindrop-size distributions | 9814 | | 15 764 | | 8473 | |
| Range of rainfall rates | 0·1 to 47·9 | | 0·1 to 35·2 | | 0·1 to 34·4 | |
| Correlation coefficient | 0·940 | | 0·936 | | 0·934 | |
| $A$ | 249 | 206 | 258 | 206 | 241 | 197 |
| Standard deviation of $A$ | 23 | 16 | 36 | 14 | 26 | 18 |
| $B$ | 1·74 | 1·53 | 1·85 | 1·60 | 1·76 | 1·52 |
| Standard deviation of $B$ | 0·026 | 0·021 | 0·111 | 0·058 | 0·075 | 0·020 |

It can be seen from Table III that the $Z$–$R$ relationships for the three years are very similar, approximating to 250 $R^{1·8}$ for a log $R$ on log $Z$ regression, and 200 $R^{1·55}$ for a log $Z$ on log $R$ regression. (It was noted that about 1000 minutes were needed to obtain a consistent $Z$–$R$ relationship in each year.)

There is considerable doubt which form of regression some past authors have used. The values of $A$ and $B$ in the $Z = AR^B$ relationship for North Wales calculated by a log $Z$ on log $R$ regression are very similar to those calculated by Marshall and Palmer (i.e. $Z = 220\ R^{1·6}$). However, calculating $A$ and $B$ from a log $R$ on log $Z$ regression results in higher values than those obtained by most other authors who used this form of regression, though theirs are based on a relatively small number of drop-size spectra when compared with the very large number of distributions shown in Table III.

It is apparent that, in order to improve the accuracy of measuring rainfall by radar methods over North Wales, a value of 1·8 for the exponent in the $Z$–$R$ relationship should replace the value of 1·6 which is used at present (see Harrold *et alii*.)

A comparison of results has shown that using a value of 1·8 the improvement in accuracy at low rainfall rates ($<$1·0 mm $h^{-1}$) is about 10 per cent. At rainfall rates of most interest (i.e. $\geqslant$1 mm $h^{-1}$) only a small improvement of 2–3 per cent is achieved.

The lack of marked improvement is attributed to the fact that the radar samples aloft (about 300 m above ground at Carrog and 500 m above ground at Bala) whereas the relationships calculated in this paper are derived from drop-size distributions measured at ground level, which may have been affected by changes during the fall of the drops caused by low-level growth or evaporation, and size sorting by wind shears.

It has been proposed (Joss *et alii* (1970)) that more accurate measurements of rainfall can be made using radar if a different $Z$–$R$ relationship is used for

various synoptic weather types. Table IV shows the average $Z$–$R$ relationships in the three summers for five synoptic weather types, together with the number of drop-size distributions used and the range of rainfall rates.

TABLE IV—RADAR REFLECTIVITY ($Z$) AND RAINFALL RATE ($R$) RELATIONSHIPS (FROM LOG $R$ ON LOG $Z$ REGRESSION) FOR VARIOUS SYNOPTIC TYPES($Z=AR^B$)

| | Carrog 1972 | | Bala 1973 | | Carrog 1974 | |
|---|---|---|---|---|---|---|
| | *A* | *B* | *A* | *B* | *A* | *B* |
| Cold fronts | 224 | 1·72 | 268 | 1·84 | 220 | 1·90 |
| Number of distributions | 2477 | | 3814 | | 1736 | |
| Maximum rainfall rate* | 12·3 | | 7·4 | | 11·1 | |
| Warm fronts | 264 | 1·60 | 229 | 1·79 | 260 | 1·66 |
| Number of distributions | 778 | | 2257 | | 1126 | |
| Maximum rainfall rate | 12·3 | | 25·4 | | 5·7 | |
| Occlusions | 422 | 2·01 | 194 | 1·95 | 316 | 1·65 |
| Number of distributions | 211 | | 805 | | 1175 | |
| | (only one storm) | | | | | |
| Maximum rainfall rate | 2·0 | | 12·8 | | 15·1 | |
| Cyclonic rains | 214 | 1·68 | 232 | 1·70 | 193 | 1·64 |
| Number of distributions | 1408 | | 2613 | | 1886 | |
| Maximum rainfall rate | 44·7 | | 24·9 | | 21·4 | |
| Showers | 234 | 1·70 | 254 | 1·79 | 235 | 1·64 |
| Number of distributions | 1320 | | 942 | | 1716 | |
| Maximum rainfall rate | 47·9 | | 35·2 | | 34·4 | |

* The minimum rainfall rate is 0·1 mm $h^{-1}$.

The results show that by reducing the data samples by subdivision there is a greater uncertainty in the $Z$–$R$ relationship, and that any increase in accuracy which might be achieved would not justify the extra effort involved.

An attempt was made to relate the 'constants' $A$ and $B$ for the different synoptic types to low- and medium-level wind velocity but little correlation was established.

Table V shows the storm-to-storm variation of $Z$–$R$ relationships, using Carrog 1972 as an example. A few storms with very small ranges of rainfall rates have been excluded.

It can be seen that if only a few storms, or parts of storms, had been used as the data sample on which to base the $Z$–$R$ relationships a large error would be possible. Furthermore, if the mean $Z$–$R$ relationship were to be applied to radar rainfall measurements, considerable day-to-day inaccuracy would result unless the radar system was additionally calibrated using a few widely spaced telemetering rain-gauges in the areas of interest (see Harrold *et alii*). The use of a number of distrometers to provide real-time $Z$–$R$ relationships as an input to the radar would not seem to be feasible, because of the large amount of data processing involved.

Mean relationships between $Z$ and $R$ for all synoptic types for the three summers are illustrated in Figure 7.

## The relationship of attenuation to rate of rainfall

Many authors (see for example Harrold (1974)) have stressed the importance of attenuation, caused by precipitation particles, on the transmission of radio signals at millimetric wavelengths. The ability of radar to measure precipitation is affected to a lesser extent by attenuation but nevertheless must be considered.

Gunn and East (1954) comprehensively reviewed the physics of attenuation and showed that it was derived from a complicated function dependent on the wavelength of the radiation, the temperature and the drop-size distribution.

TABLE V—STORM RADAR REFLECTIVITY ($Z$) AND RAINFALL RATE ($R$) RELATIONSHIPS AT CARROG 1972

| Date | Number of drop-size distributions | $Z$–$R$ relation | | Correlation coefficient | Range of rainfall rates (mm h$^{-1}$) |
|---|---|---|---|---|---|
| | | $A$ | $B$ | | |
| Cold fronts | | | | | |
| 29 Apr. | 262 | 232 | 1·93 | 0·925 | 0·1– 3·4 |
| 25 May | 62 | 197 | 1·48 | 0·964 | 0·1– 3·0 |
| 2 June | 287 | 259 | 1·85 | 0·949 | 0·1– 6·2 |
| 8 June | 60 | 258 | 1·51 | 0·976 | 0·1– 2·3 |
| 27 June | 72 | 566 | 1·91 | 0·940 | 0·1– 3·3 |
| 4 Aug. | 197 | 279 | 1·92 | 0·944 | 0·1–11·0 |
| 5 Aug. | 37 | 92 | 1·65 | 0·977 | 0·1– 2·5 |
| 8 Aug. | 73 | 114 | 1·31 | 0·980 | 0·1–12·3 |
| 17 Aug. | 96 | 89 | 1·58 | 0·959 | 0·1–12·0 |
| 6 Sept. | 188 | 191 | 1·78 | 0·945 | 0·1– 8·5 |
| 8 Sept. | 324 | 190 | 1·61 | 0·906 | 0·1– 6·3 |
| 11 Oct. | 414 | 231 | 1·64 | 0·944 | 0·1– 7·0 |
| Warm fronts | | | | | |
| 28 Apr. | 286 | 296 | 1·75 | 0·952 | 0·1– 3·6 |
| 2 June | 71 | 216 | 1·32 | 0·975 | 0·1– 2·3 |
| 7 Aug. | 205 | 225 | 1·43 | 0·962 | 0·1–12·3 |
| 8 Sept. | 142 | 379 | 1·59 | 0·967 | 0·1– 3·1 |
| Occlusions | | | | | |
| 20 June | 211 | 422 | 2·01 | 0·906 | 0·1– 2·0 |
| Cyclonic rains | | | | | |
| 26 May | 438 | 272 | 1·65 | 0·959 | 0·1– 9·3 |
| 9 June | 255 | 320 | 1·52 | 0·961 | 0·1–44·7 |
| 1 Aug. | 347 | 173 | 1·51 | 0·970 | 0·1–31·2 |
| 8 Aug. | 368 | 119 | 1·51 | 0·958 | 0·1– 8·8 |
| Showers and thunderstorms | | | | | |
| 30 May | 407 | 232 | 1·65 | 0·953 | 0·1–21·6 |
| 1 June | 139 | 224 | 1·55 | 0·967 | 0·1–47·9 |
| 27 June | 223 | 187 | 1·83 | 0·917 | 0·1– 4·4 |
| 6 July | 67 | 293 | 1·76 | 0·903 | 0·1– 2·4 |
| 29 July | 29 | 349 | 1·59 | 0·985 | 0·1–16·7 |
| 30 July | 61 | 487 | 1·51 | 0·961 | 0·1– 3·5 |
| 9 Aug. | 45 | 241 | 1·71 | 0·948 | 0·1–16·1 |
| 13 Aug. | 68 | 165 | 1·59 | 0·970 | 0·1– 3·3 |
| 7 Sept. | 91 | 204 | 1·72 | 0·924 | 0·1– 3·8 |
| 13 Sept. | 164 | 237 | 1·71 | 0·953 | 0·1–11·7 |

They showed that at wavelengths which are long compared with the drop diameter the one-way attenuation ($K$), (i.e. half the total attenuation caused by precipitation *en route* to target and on return), in decibels per kilometre is given by

$$K = 0{\cdot}4343\,\Sigma Q_t,$$

where the summation is over one cubic metre and $Q_t$ (the total absorption coefficient) is in square centimetres. Herman *et alii* (1961) have produced tables of normalized attenuation at 0°C ($=4Q_t/\pi D^2$) for a variety of wavelengths and drop sizes. From this quantity it is possible to calculate the attenuation caused by the measured drop-size distribution and hence to relate this to the rate of rainfall or radar reflectivity, both of which are derived from the distribution.

For each of six wavelengths frequently used in radar or radio work, the attenuation has been calculated in each minute when the rate of rainfall was 0·1 mm h$^{-1}$ or more. A mean relationship of the type

$$K = ER^F$$

has been found by a least-squares linear regression of log $K$ on log $R$. Table VI

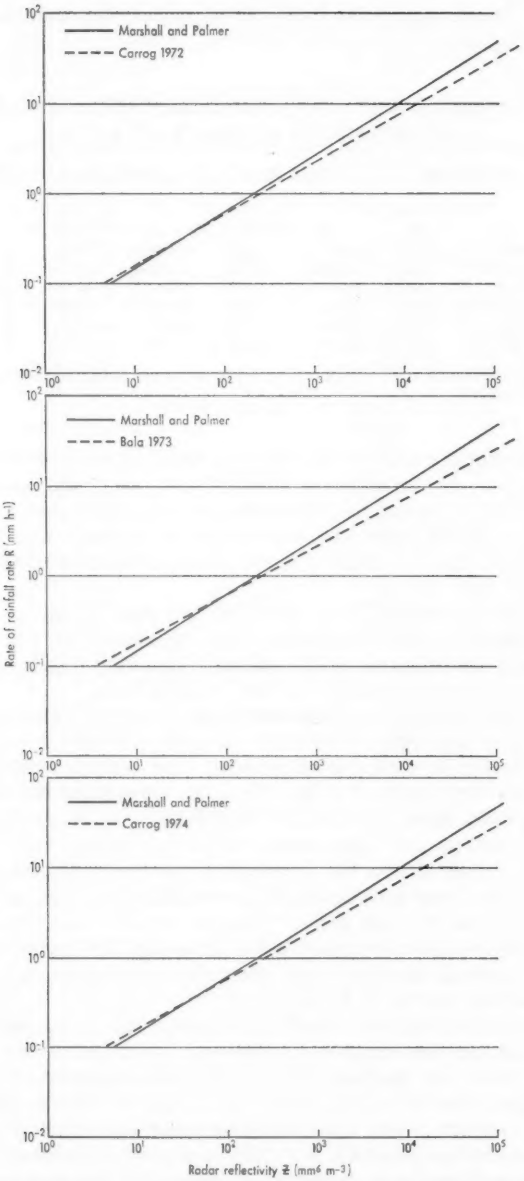


**FIGURE 7—THE MEAN RELATIONSHIPS BETWEEN RATE OF RAINFALL AND RADAR REFLECTIVITY FACTOR FOR THE THREE SUMMERS**

shows the results of these regression calculations and a comparison with those calculated by Wexler and Atlas (1963). In every case the correlation coefficient was 0·97 or higher.

TABLE VI—ATTENUATION ($K$) AND RAINFALL RATE ($R$) RELATIONSHIPS BASED ON ATTENUATION CONSTANTS $E$,$F$ AT 0°C

| Wavelength *cm* | Carrog 1972 | | Bala 1973 | | Carrog 1974 | | Marshall–Palmer after Wexler–Atlas | |
|---|---|---|---|---|---|---|---|---|
| | *E* | *F* | *E* | *F* | *E* | *F* | *E* | *F* |
| 10·0 | 0·00071 | 0·98 | 0·00075 | 0·95 | 0·00070 | 0·99 | 0·00082 | 1·0 |
| 5·5 | 0·0026 | 0·94 | 0·0028 | 0·89 | 0·0026 | 0·92 | 0·0031 | 1·0 |
| 3·21 | 0·011 | 1·01 | 0·011 | 0·97 | 0·011 | 0·98 | 0·013 | 1·15 |
| 1·87 | 0·038 | 1·11 | 0·041 | 1·10 | 0·037 | 1·12 | 0·050 | 1·10 |
| 1·24 | 0·099 | 1·05 | 0·105 | 1·03 | 0·097 | 1·05 | 0·13 | 1·07 |
| 0·86 | 0·215 | 1·07 | 0·229 | 1·05 | 0·214 | 1·06 | 0·31 | 1·0 |

The attenuation constants were computed by Herman *et alii* using the Marshall and Palmer drop-size spectrum.

It can be seen that in each year the two 'constants' in the $K$–$R$ relationships are very similar. At the wavelengths usually used for radar measurement of precipitation (5·5 to 10 cm) the attenuation is very small even at high rainfall rates. Using a 5·5-cm radar the attenuation at rainfall rates of 1 mm $h^{-1}$ and 5 mm $h^{-1}$ which are typical of North Wales result in attenuation of 0·0026 dB $km^{-1}$ and 0·0118 dB $km^{-1}$.

The relationships derived from each year are also similar to those derived using a Marshall–Palmer distribution. The differences are attributed to the mean drop-size distributions described earlier showing slight variations from the Marshall–Palmer distribution at most rainfall rates.

The $K$–$R$ relationships have been calculated using the full range of rainfall rates, although a high proportion of the data are for relatively low rates. For example, at Carrog in 1972, out of the 9814 drop spectra used, 9565 were for rates of rainfall of 5·0 mm $h^{-1}$ or less. Thus the mean attenuation relationships obtained are most applicable at low rainfall rates, although the attenuation is more important at the higher rates, which fortunately occur infrequently. In an attempt to show how the attenuation relation is dependent on rainfall rate, the data have been divided into three overlapping ranges of rainfall rates, namely 0·1–5·0, 1·0–10·0 and greater than 5·0 mm $h^{-1}$, and the $K$–$R$ relationships calculated by linear regression for drop-size distributions within these ranges. The results in the three years are very similar again, so only those for Carrog in 1972 are shown in Table VII.

Table VII shows that at wavelengths of 10 cm and 0·86 cm there is little to choose between the three regression equations. For the other wavelengths it can be seen that there is a tendency for the regression equations to have steeper slopes at higher rainfall rates, implying that the attenuation may become a more serious problem at very high rates than would be suggested by the original relationships shown in Table VI. A very large number of drop-size distributions are required, particularly at higher rates before the attenuation relationship can be defined with confidence at the rates of rainfall when it is more important.

Empirical relationships between attentuation ($K$) and radar reflectivity ($Z$) can be derived from the previously calculated $Z$–$R$ and $K$–$R$ relationships.

For example, at Carrog in 1972 for a wavelength of 5·5 cm

$$Z = 249R^{1.74} \text{ and } K = 0.0026R^{0.94}$$

lead to the relationship

$$K = 1.32 \times 10^{-4} Z^{0.54}.$$

TABLE VII—ATTENUATION ($K$) AND RAINFALL RATE ($R$) RELATIONSHIPS OF FORM $K=ER^F$ AS A FUNCTION OF $R$ AT CARROG 1972

| Wavelength | Range of rainfall rates | Number of distributions | $K$–$R$ relationship | | Correlation coefficient |
|---|---|---|---|---|---|
| *cm* | *mm h*$^{-1}$ | | *E* | *F* | |
| 10·0 | 0·1– 5·0 | 9565 | 0·00071 | 0·98 | 0·978 |
| | 1·0–10·0 | 2575 | 0·00072 | 0·90 | 0·971 |
| | More than 5·0 | 249 | 0·00067 | 0·95 | 0·992 |
| 5·5 | 0·1– 5·0 | | 0·0026 | 0·93 | 0·973 |
| | 1·0–10·0 | As above | 0·0025 | 0·95 | 0·976 |
| | More than 5·0 | | 0·0020 | 1·10 | 0·973 |
| 3·21 | 0·1– 5·0 | | 0·0106 | 1·00 | 0·980 |
| | 1·0–10·0 | As above | 0·0102 | 1·06 | 0·974 |
| | More than 5·0 | | 0·0074 | 1·23 | 0·956 |
| 1·87 | 0·1– 5·0 | | 0·0383 | 1·11 | 0·985 |
| | 1·0–10·0 | As above | 0·0380 | 1·09 | 0·978 |
| | More than 5·0 | | 0·0317 | 1·18 | 0·974 |
| 1·24 | 0·1– 5·0 | | 0·0983 | 1·05 | 0·986 |
| | 1·0–10·0 | As above | 0·0966 | 1·07 | 0·983 |
| | More than 5·0 | | 0·0815 | 1·16 | 0·980 |
| 0·86 | 0·1– 5·0 | | 0·216 | 1·07 | 0·987 |
| | 1·0–10·0 | As above | 0·215 | 1·06 | 0·986 |
| | More than 5·0 | | 0·209 | 1·07 | 0·990 |

## OTHER METEOROLOGICAL PARAMETERS DERIVED FROM DROP-SIZE DISTRIBUTIONS

Although they are not frequently used in connection with radar rainfall measurements two other parameters, the liquid water content ($M$) and the median-volume drop diameter ($D_0$) can be calculated from the drop-size distribution.

The liquid water content is the mass of water (in grams) contained in 1 m$^3$ and is calculated by the summation of the masses of water contained in each drop category in 1 m$^3$;

$$\text{that is } M = \sum_{1\text{m}^3} N_D M_D \, \Delta D,$$

where $M_D$ is the mass of a drop with a diameter in the range of $D$ to $D + \Delta D$.

The liquid water content is related to the rate of rainfall by an empirical relationship of the type $M = GR^H$. The 'constants' can be derived by a least-squares linear regression of log $M$ on log $R$, as in previous sections.

The liquid water content has been calculated for each drop-size distribution in the three years and the results of the linear regressions are shown in Table VIII. Also shown are the 'constants' in the relationship $Z = JM^P$ which can be derived from the $Z$–$R$ and $M$–$R$ relationships.

The median-volume drop diameter ($D_0$) has been defined by Atlas (1953) as that drop diameter which divides the drop distribution in such a way that half the liquid water content is contained in drops greater than $D_0$. As the liquid water content is related to the rate of rainfall by an empirical power relationship as above, it is reasonable to seek a similar relationship between the median-

TABLE VIII—LIQUID WATER CONTENT ($M$), RAINFALL RATE ($R$) AND RADAR REFLECTIVITY ($Z$) RELATIONSHIPS

| Site | Number of distributions | $M=GR^H$ relationship (log $M$ on log $R$) | | Correlation coefficient | $Z=JM^P$ relationship (direct calculation) | |
|---|---|---|---|---|---|---|
| | | $G$ | $H$ | | $J$ | $P$ |
| Carrog 1972 | 9 814 | 0·0794 | 0·93 | 0·960 | $2{\cdot}79\times10^4$ | 1·86 |
| Bala 1973 | 15 764 | 0·0863 | 0·90 | 0·946 | $3{\cdot}93\times10^4$ | 2·05 |
| Carrog 1974 | 8 473 | 0·0805 | 0·93 | 0·958 | $2{\cdot}87\times10^4$ | 1·90 |

volume drop diameter and rainfall rate. The results of log $D_0$ on log $R$ regressions for the three years' data are shown in Table IX. Atlas reported that the scatter about a line of the form $D_0 = SR^T$ (where $D_0$ is in mm and $R$ in mm $h^{-1}$) is fairly large, and this is demonstrated by the rather low correlation coefficients in the Table.

TABLE IX—MEDIAN-VOLUME DROP DIAMETER ($D_0$) AND RAINFALL RATE ($R$) RELATIONSHIPS

| Site | Number of distributions | $D_0=SR^T$ relationship | | Correlation coefficient |
|---|---|---|---|---|
| | | $S$ | $T$ | |
| Carrog 1972 | 9 814 | 0·994 | 0·20 | 0·630 |
| Bala 1973 | 15 764 | 0·961 | 0·24 | 0·677 |
| Carrog 1974 | 8 473 | 0·962 | 0·21 | 0·623 |

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# THE HEIGHT OF THE FREEZING LEVEL DURING RAINFALL OVER THE BRITISH ISLES

By C. G. COLLIER

SUMMARY

An analysis of the height of the freezing level over the British Isles is presented for November, January and March using the ten years of data 1964–73. The relationship of the spatial and temporal variations of this parameter to twelve-hour rainfall amount is investigated.

## 1. INTRODUCTION

Recent work on the measurement of rainfall using radar (see for example Harrold *et alii* (1974)) has demonstrated that radar rainfall measurements may be made with an accuracy of 15 to 20 per cent over periods of three hours, provided that the melting level is not intersected by the radar beam. When the melting level intersects the radar beam the accuracy is significantly decreased, since the presence of melting snow in the beam causes the power received at the aerial to be greater than that from rain at the same rate of precipitation (see Battan (1973)). Methods which attempt to correct the radar rainfall estimates when this occurs have been developed (see Harrold and Kitchingman (1975)). Nevertheless the errors that the melting level may introduce into the radar estimates cannot be completely removed. In order to examine how often this error may affect radar measurements in any part of the British Isles, a study is presented of the spatial variation of the height of the freezing level meaned over a ten-year period. The study is only concerned with the 0°C level. The centre of the melting layer is below this height, occasionally by several hundred metres (see Battan (1973)).

Previous work on the height of the freezing level by Murray (1950) has indicated its seasonal variations over the British Isles using six years of data 1942–47. The results are not related to rainfall, and therefore not directly applicable to the estimation of frequency of error occurrence due to melting-level effects in radar rainfall measurements.

In processing the data, when the temperature at the surface was less than 0°C the freezing level was taken as being at the surface. For cases in which a sounding had two or more heights at which the 0°C isotherm was intersected, only the lower height was considered, as this height is closest to the true height of the melting level relevant to radar measurement accuracy, as explained above. The freezing-level heights have been extracted from the radiosonde data recorded twice daily (00 and 12 GMT) at each of the routinely manned British Isles upper-air stations. The rainfall data have been extracted from the *Daily Weather Reports* for surface stations within 25 km of the upper-air stations. Twelve-hour rainfall totals spanning the times of the radiosonde ascents (the 'rainday' begins at 09 GMT) have been used in the analysis. Since the atmospheric structure varies considerably with time, the use of one ascent to represent 12 hours of rain imposes a severe limitation on the analysis, but more frequent upper-air temperature data are not available as routine.

## 2. Spatial variation of freezing-level height in winter

The variation of freezing-level height over the British Isles has been examined by Murray (1950) for each month of the year covering the period 1942–47. The ten years of data 1964–73 considered in the present study give mean freezing-level heights, use being made of all the data for the months of November, January and March, which agree well with those given in the earlier study. During the winter months the mean height of the freezing level, all data being considered without regard to rainfall amount in 12 hours, decreases uniformly from approximately 1100 metres over the south-west of the British Isles to approximately 600 metres over the northern tip of the Shetlands, Figure 1. In summer this lowering of freezing-level height is from the south-east to the north-east (see Murray (1950)), and the transition from summer to winter is illustrated by the month of November shown in Figure 1. The standard deviations about the mean values are between $\pm 300$ metres and $\pm 800$ metres, see Figure 2.

It is of interest to potential users of a radar system measuring rainfall to examine whether or not these mean maps are appropriate for all rainfall rates. The lower the height of the freezing level, the closer to the radar site its intersection with the beam, and therefore the greater the area of radar coverage affected by measurement errors introduced by its presence. Several ranges of 12-hour rainfall amount have been considered.

### 2.1 *Rainfall rate zero or 0·05 mm in 12 hours*

The distributions of mean freezing-level heights for zero or negligible rainfall in 12 hours are shown in Figure 3. Compared with the mean distribution for all rainfall rates (Figure 1), the freezing level is higher over the south-west of the British Isles in November, lower over the north-east of the British Isles in January, and slightly higher over the south-east of the British Isles in March. Table I gives the percentage frequency of occurrence of the freezing level below specified heights for three rainfall-rate ranges. This Table gives an indication of the amount of data available for each rainfall-rate classification. About 50 per cent of the data lie within this rainfall-rate range (i.e. 0–0·05 mm/12 hours).

### 2.2 *Rainfall rates between 0·05 mm in 12 hours and 5 mm in 12 hours*

In November and March the variation of mean freezing-level height for this class (Figure 4) resembles quite closely the mean pattern for all rainfall rates, but in January it is significantly different, showing a marked rise from the south. In January over Cornwall the mean height of the freezing level for this rainfall rate range is about 200 metres higher than given by the mean pattern (Figure 1), and this height increase is reflected throughout the British Isles. Approximately 45 per cent of the data fall within this rainfall-rate range (Table I).

### 2.3 *Rainfall rates between 5 mm in 12 hours and 10 mm in 12 hours*

Figure 5 shows a lowering in the mean height of the freezing level for this rainfall-rate range in both November and March compared with the distribution for all rainfall rates (Figure 1), in November from the north-west, and in March from the north over the eastern part of the British Isles. This effect is most marked in March when it produces a strong gradient of freezing-level height from east to west over England, and from north-east to south-west over Scotland. In March the mean freezing-level heights for this rainfall-rate range are 50 per cent lower than the mean values for all rainfall rates (Figure 1) over the

TABLE I—PERCENTAGE PROBABILITY OF THE 0°C LEVEL BEING BELOW SPECIFIED HEIGHTS ($H_m$) FOR RAINFALL AT SPECIFIED RATES ($R$ mm/12 HOURS) DURING NOVEMBER, JANUARY AND MARCH

| | $H_m$ metres | 500 | | | 1000 | | | 1500 | | | 2000 | | | 2500 | | | 3000 | | | 3500 | | | 4000 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $R$ mm/12 h | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 | 0/0·05 | 0·05–5 | 5–10 |
| November | Camborne | 0 | 1 | 0 | 10 | 11 | 2 | 18 | 24 | 4 | 22 | 31 | 5 | 27 | 36 | 6 | 35 | 41 | 7 | 40 | 46 | 7 | 41 | 47 | 7 |
| | Valentia | 1 | 0 | 0 | 7 | 19 | 2 | 15 | 35 | 4 | 17 | 41 | 4 | 22 | 47 | 6 | 27 | 52 | 7 | 32 | 54 | 7 | 32 | 55 | 7 |
| | Crawley | 9 | 3 | 0 | 25 | 8 | 1 | 35 | 17 | 2 | 39 | 21 | 3 | 44 | 25 | 4 | 54 | 29 | 4 | 61 | 31 | 4 | 63 | 31 | 4 |
| | Hemsby | 4 | 4 | 1 | 20 | 18 | 2 | 30 | 28 | 3 | 34 | 32 | 3 | 41 | 35 | 4 | 50 | 38 | 4 | 55 | 39 | 4 | 55 | 39 | 4 |
| | Aughton | 7 | 2 | 0 | 24 | 15 | 2 | 34 | 25 | 3 | 38 | 30 | 4 | 43 | 34 | 4 | 50 | 37 | 5 | 54 | 38 | 5 | 54 | 39 | 5 |
| | Long Kesh | 5 | 6 | 1 | 22 | 22 | 3 | 28 | 32 | 3 | 31 | 38 | 4 | 36 | 44 | 5 | 41 | 46 | 5 | 44 | 48 | 5 | 44 | 48 | 5 |
| | Shanwell | 14 | 4 | 1 | 37 | 17 | 1 | 47 | 23 | 2 | 51 | 27 | 2 | 55 | 30 | 2 | 62 | 31 | 2 | 64 | 32 | 2 | 64 | 32 | 2 |
| | Stornoway | 3 | 12 | 1 | 15 | 41 | 4 | 20 | 52 | 6 | 22 | 55 | 7 | 25 | 58 | 8 | 28 | 61 | 8 | 28 | 62 | 8 | 28 | 62 | 8 |
| | Lerwick | 8 | 15 | 2 | 15 | 46 | 6 | 18 | 55 | 7 | 20 | 58 | 8 | 22 | 60 | 9 | 23 | 62 | 9 | 24 | 63 | 9 | 24 | 63 | 9 |
| January | Camborne | 7 | 1 | 0 | 22 | 16 | 1 | 30 | 29 | 3 | 33 | 37 | 6 | 37 | 42 | 7 | 42 | 45 | 7 | 44 | 45 | 7 | 44 | 45 | 7 |
| | Valentia | 7 | 1 | 0 | 20 | 17 | 2 | 31 | 33 | 4 | 33 | 39 | 6 | 35 | 44 | 7 | 37 | 46 | 8 | 38 | 47 | 8 | 38 | 47 | 8 |
| | Crawley | 20 | 7 | 0 | 36 | 16 | 1 | 43 | 27 | 2 | 48 | 34 | 2 | 52 | 37 | 3 | 57 | 38 | 3 | 58 | 39 | 3 | 58 | 39 | 3 |
| | Hemsby | 19 | 10 | 0 | 36 | 20 | 1 | 44 | 31 | 1 | 50 | 36 | 1 | 54 | 39 | 1 | 57 | 41 | 1 | 58 | 41 | 1 | 58 | 41 | 1 |
| | Aughton | 21 | 5 | 0 | 35 | 18 | 1 | 45 | 28 | 3 | 50 | 32 | 4 | 54 | 35 | 5 | 58 | 35 | 5 | 59 | 35 | 5 | 59 | 35 | 5 |
| | Long Kesh | 14 | 7 | 1 | 30 | 23 | 2 | 37 | 36 | 3 | 41 | 41 | 4 | 44 | 44 | 5 | 47 | 46 | 5 | 47 | 46 | 5 | 47 | 46 | 5 |
| | Shanwell | 19 | 10 | 1 | 36 | 24 | 3 | 45 | 33 | 4 | 49 | 36 | 4 | 54 | 38 | 4 | 56 | 38 | 4 | 56 | 38 | 4 | 56 | 38 | 4 |
| | Stornoway | 9 | 11 | 1 | 25 | 34 | 3 | 34 | 49 | 4 | 36 | 53 | 4 | 37 | 55 | 5 | 38 | 55 | 5 | 38 | 55 | 5 | 38 | 55 | 5 |
| | Lerwick | 8 | 22 | 1 | 18 | 50 | 4 | 21 | 59 | 5 | 23 | 63 | 5 | 24 | 65 | 6 | 25 | 66 | 6 | 25 | 66 | 6 | 25 | 66 | 6 |
| March | Camborne | 8 | 2 | 0 | 29 | 14 | 1 | 41 | 23 | 2 | 50 | 31 | 3 | 55 | 33 | 3 | 60 | 34 | 3 | 62 | 34 | 3 | 62 | 34 | 3 |
| | Valentia | 3 | 1 | 0 | 23 | 22 | 1 | 34 | 35 | 2 | 40 | 42 | 3 | 45 | 43 | 3 | 49 | 44 | 3 | 50 | 44 | 3 | 50 | 44 | 3 |
| | Crawley | 17 | 3 | 1 | 38 | 12 | 1 | 54 | 20 | 1 | 60 | 24 | 2 | 67 | 25 | 3 | 69 | 25 | 3 | 71 | 25 | 3 | 72 | 25 | 3 |
| | Hemsby | 18 | 9 | 1 | 38 | 22 | 1 | 51 | 28 | 2 | 56 | 31 | 2 | 62 | 32 | 2 | 62 | 32 | 2 | 66 | 32 | 2 | 66 | 32 | 2 |
| | Aughton | 17 | 6 | 1 | 39 | 18 | 2 | 51 | 26 | 3 | 59 | 30 | 4 | 61 | 30 | 4 | 63 | 31 | 4 | 65 | 31 | 4 | 65 | 31 | 4 |
| | Long Kesh | 10 | 9 | 1 | 32 | 25 | 2 | 44 | 34 | 3 | 49 | 39 | 4 | 51 | 40 | 4 | 54 | 40 | 4 | 55 | 40 | 4 | 55 | 40 | 4 |
| | Shanwell | 20 | 9 | 1 | 46 | 21 | 1 | 58 | 25 | 1 | 63 | 27 | 1 | 68 | 28 | 1 | 70 | 28 | 1 | 71 | 28 | 1 | 71 | 28 | 1 |
| | Stornoway | 10 | 12 | 1 | 28 | 36 | 4 | 36 | 45 | 5 | 39 | 50 | 5 | 41 | 51 | 6 | 42 | 51 | 6 | 42 | 51 | 6 | 42 | 51 | 6 |
| | Lerwick | 14 | 21 | 4 | 29 | 41 | 5 | 35 | 49 | 7 | 36 | 52 | 7 | 37 | 53 | 8 | 37 | 53 | 8 | 37 | 53 | 8 | 37 | 53 | 8 |

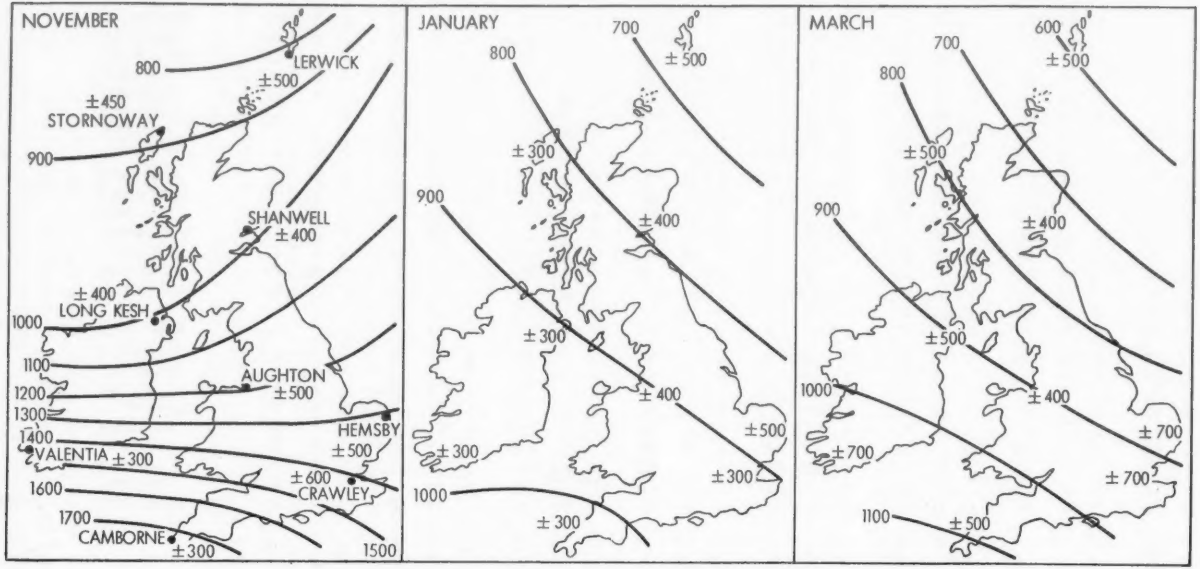


**FIGURE I—MEAN HEIGHT OF FREEZING LEVEL OVER THE BRITISH ISLES IN METRES FOR NOVEMBER, JANUARY AND MARCH 1964–73**

The positions of the radiosonde stations used in the study are also shown, with an indication of the standard deviations of the heights at each station.

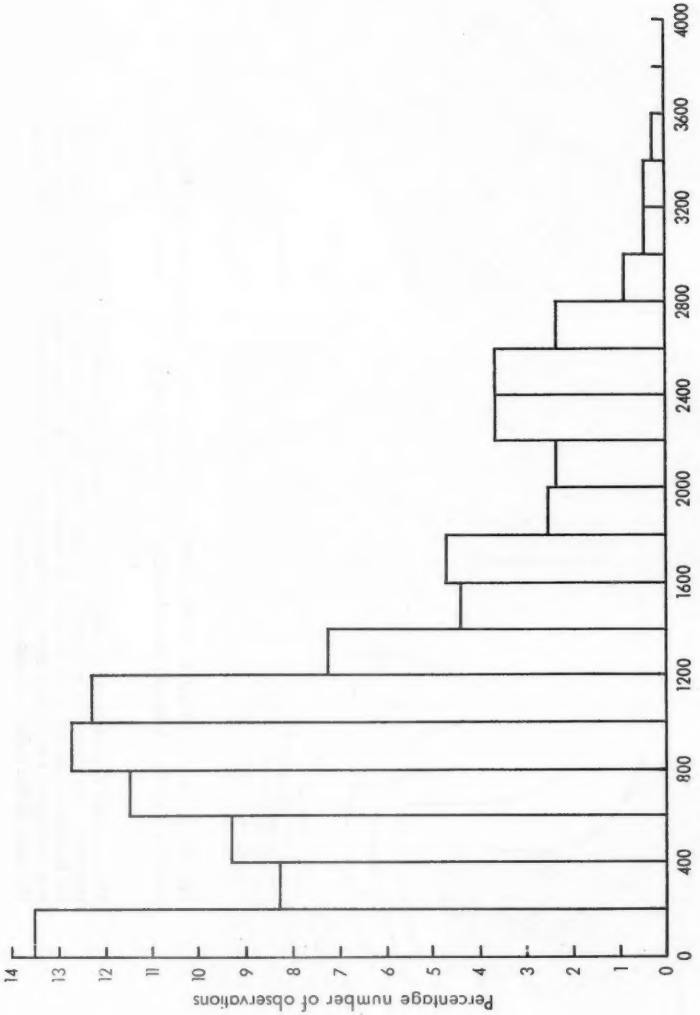


FIGURE 2—PERCENTAGE FREQUENCY OF HEIGHT OF FREEZING LEVEL IN JANUARY IN INTERVALS OF 200 m FOR AUGHTON 1964–73

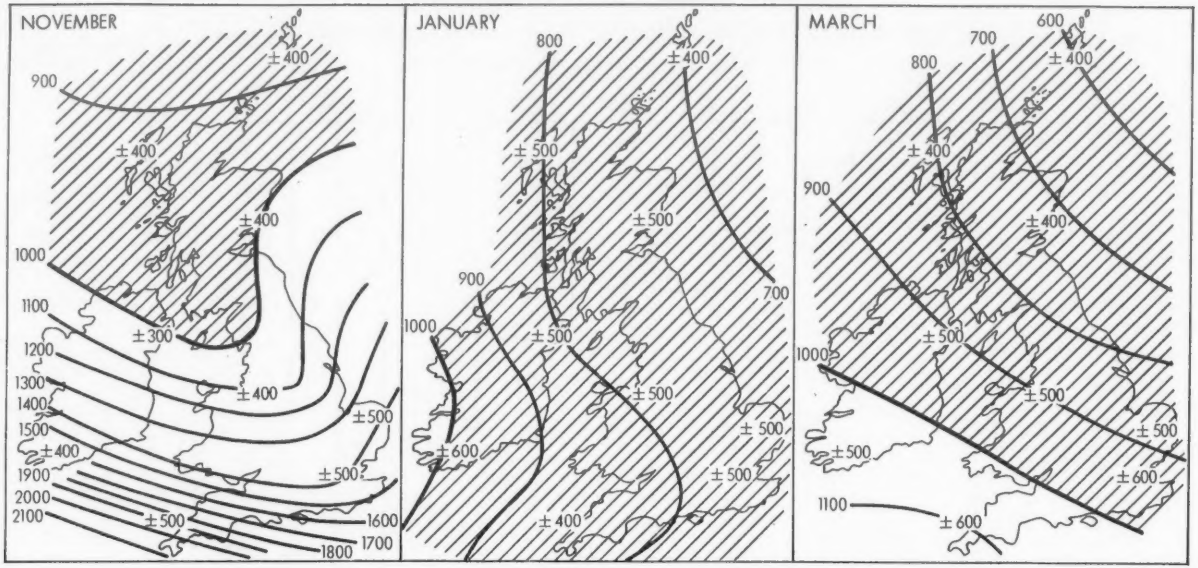


**FIGURE 3—MEAN HEIGHT OF FREEZING LEVEL OVER THE BRITISH ISLES IN METRES FOR NOVEMBER, JANUARY AND MARCH FOR ZERO RAINFALL OR A TRACE (0·05 mm) IN 12 HOURS**

The standard deviations of the heights at each radiosonde station are indicated. For a radar situated 100 m above sea level with a 1° wide beam and the beam axis elevated by 0·5° from the horizontal, the 0°C level intersects the beam at about 50 km range from the radar when it is at a height of 1 km. The stippling shows the areas within which the 0°C level would intersect the beam of this radar at ranges less than 50 km.

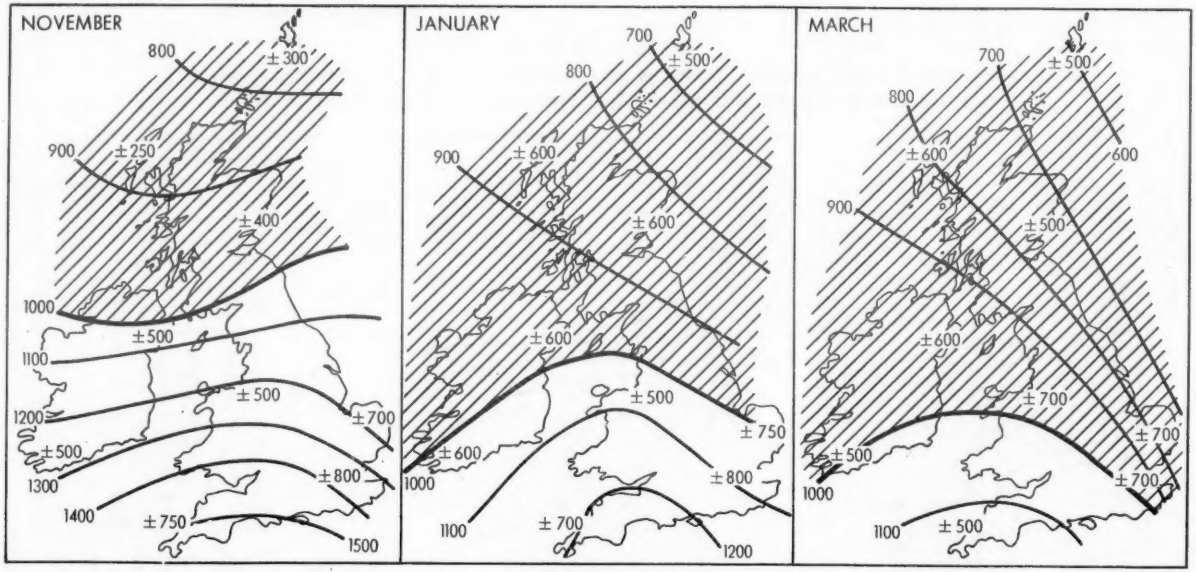


FIGURE 4—MEAN HEIGHT OF FREEZING LEVEL OVER THE BRITISH ISLES IN METRES FOR NOVEMBER, JANUARY AND MARCH FOR RAINFALL GREATER THAN A TRACE (0·05 mm) TO 5 mm IN 12 HOURS

See footnote for Figure 3.

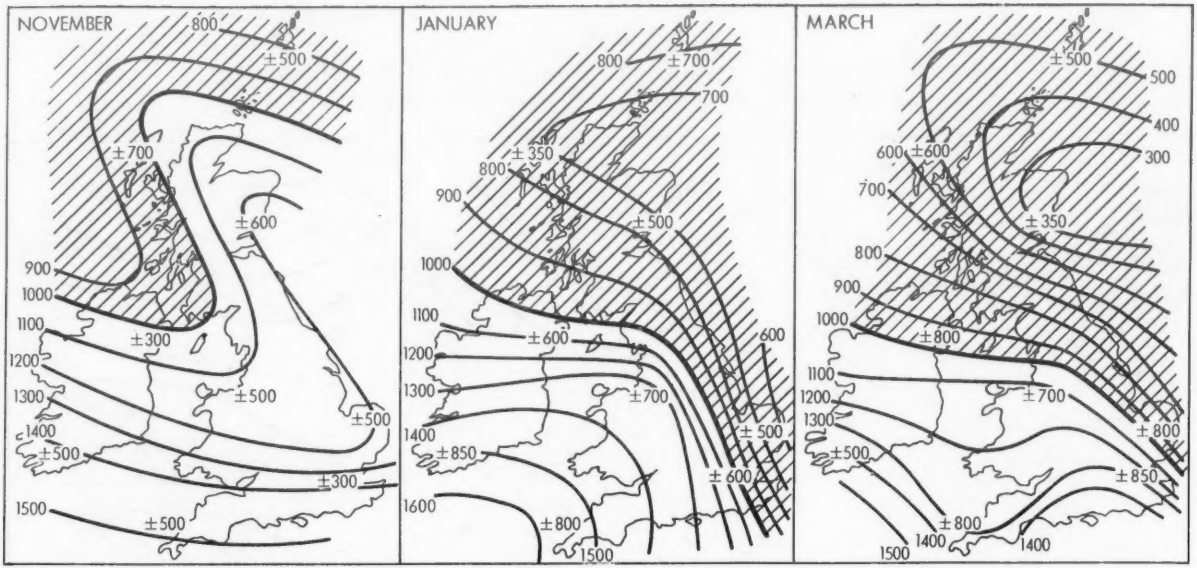


**FIGURE 5—MEAN HEIGHT OF FREEZING LEVEL OVER THE BRITISH ISLES IN METRES FOR NOVEMBER, JANUARY AND MARCH FOR RAINFALL GREATER THAN 5 mm TO 10 mm IN 12 HOURS**

See footnote for Figure 3.

eastern part of the British Isles. In January the tendency for the freezing level to rise is continued from the south-west, producing a mean pattern which changes by 700 metres across central England. Table I indicates that rainfall rates in this range make up less than 10 per cent of the data sample.

Since only a very small percentage (less than 3 per cent) of cases have rainfall rates greater than 10 mm/12 hours, it is felt that the data sample for these rainfall rates is too small to be discussed with any confidence.

## 3. Discussion

The freezing-level mean height patterns show large differences from one rainfall-rate range to another, but the significance of the changes depends upon the frequency with which the rainfall rates occur. Figure 6 shows the percentage frequency of 12-hour rainfall for Ringway Airport, Manchester (taken with the upper-air data for Aughton, Liverpool) for the period 1964–73. Fewer than 2 per cent of the data are composed of cases in which the 12-hour rainfall is greater than 10 mm, 4 per cent (of data) have rainfall in the range greater than 5 mm to 10 mm, and 60 per cent (of data) have zero rain or a trace (Section 2.3).

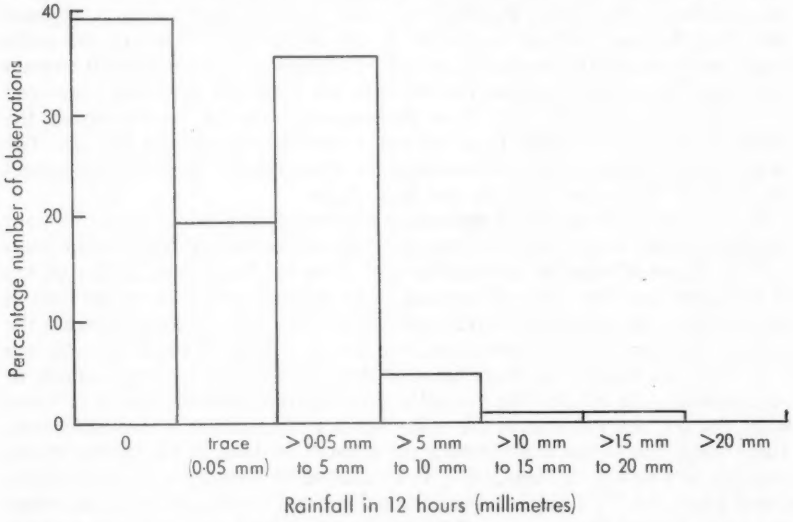


FIGURE 6—PERCENTAGE FREQUENCY OF 12-HOUR RAINFALL IN JANUARY FOR RINGWAY AIRPORT (TAKEN WITH THE UPPER-AIR DATA FOR AUGHTON), 1964–73

The marked differences from the mean patterns for the higher rainfall amounts may be related to the different frequencies of synoptic weather types through the winter months. However, there are only a small number of cases with rainfall rates in these ranges, and therefore the data sample is too small for any firm conclusions to be drawn from it. The frequency of daily weather types over the British Isles has been discussed by Lamb (1953, 1965), although the weather types are defined without reference to rainfall. Murray and Lewis (1966) extended this work, and suggested a system of indices which could be used to indicate the broad-scale circulation near the British Isles. In later papers Murray (1968, 1972) demonstrated the relationship of these indices to monthly

rainfall over England and Wales, and to monthly mean temperatures in central England. The tendency in winter months for low (high) rainfall to be associated with low (high) temperatures, and the tendency in the summer months for low (high) rainfall to be associated with high (low) temperatures were also discussed (Murray (1968)). This may account for the lowering of the freezing level in March (compared with the distribution for all rainfall rates (Figure 1) observed for the high rainfall-rate range (Figure 5) as March may be regarded as a transition month from winter to summer. The freezing level rises (compared with the distribution for all rainfall rates (Figure 1) for the high rainfall-rate ranges in January, a winter month.

It is not the purpose of this paper to seek detailed correlations between rainfall, height of the freezing level and synoptic weather type as represented by the indices by Murray and Lewis (1966). To do so would require a much more detailed discussion than has been presented in this paper. The above represents a suggestion where the explanation of the form of Figures 3 to 5 may lie.

Table I gives the percentage frequency with which the 0°C level is below a specified height, for a particular rainfall rate. The effects of latitude and season are apparent in the Table. Figures 3 to 5 and Table I may be used to estimate the mean distance, for any radar site in the British Isles, at which the radar beam intersects the freezing level, for various ranges of 12-hour rainfall amount in winter. For a radar situated 100 m above sea level with a 1° wide beam and the beam axis elevated by 0·5° from the horizontal, the 0°C level intersects the beam at about 50 km range from the radar when it is at a height of 1 km. The stippling on Figures 3 and 4 shows the areas where the 0°C level would intersect the beam of this radar at ranges less than 50 km.

Figure 7 indicates the mean percentage number of occasions with the 12-hour rainfall amount in the range 0·05 mm to 5 mm, when the 0°C level would intersect the beam of a radar, elevated by 0·5° from the horizontal, within 50 km of the radar site. The radar is assumed to be situated 100 m above sea level at each of the radiosonde sites, and at each site on the maps the upper figure is the average number of occurrences in the month with rainfall in the range 0·05 mm to 5 mm in 12 hours. The lower figure at the stations is the average number of occurrences in the month with rainfall in the range 0·05 mm to 5 mm in 12 hours when the 0°C level intersects the radar beam within 50 km of the radar site. These maps enable the mean percentage amount of data, in the rainfall range 0·05 mm to 5 mm in 12 hours, likely to be effected by measurement errors introduced when the 0°C level intersects the radar beam within 50 km of the radar site, to be derived for any part of the British Isles. Ten years of data have been used to produce Figure 7, 1964–73, and it must be stressed that any one year may differ markedly from average. For example, for the C-band Llandegla radar used in the Dee Weather radar Project (see Collier *et alii*, 1975) (400 m above sea level) in January 1974, 61 per cent of the radar data collected were affected by measurement errors introduced by the presence of the melting level in the beam within 50 km of the radar site. However, in March 1974, only 28 per cent of the radar data were affected. The corresponding mean figures derived from Figure 7 are 50 per cent in January and 60 per cent in March.

Given the frequency distribution of the rainfall-rate ranges near a particular radar site, the mean temporal variations of the range at which the radar beam intersects the 0°C level may be obtained. For example, for the Llandegla C-band radar, the data given in Table I for Aughton may be used assuming a beam

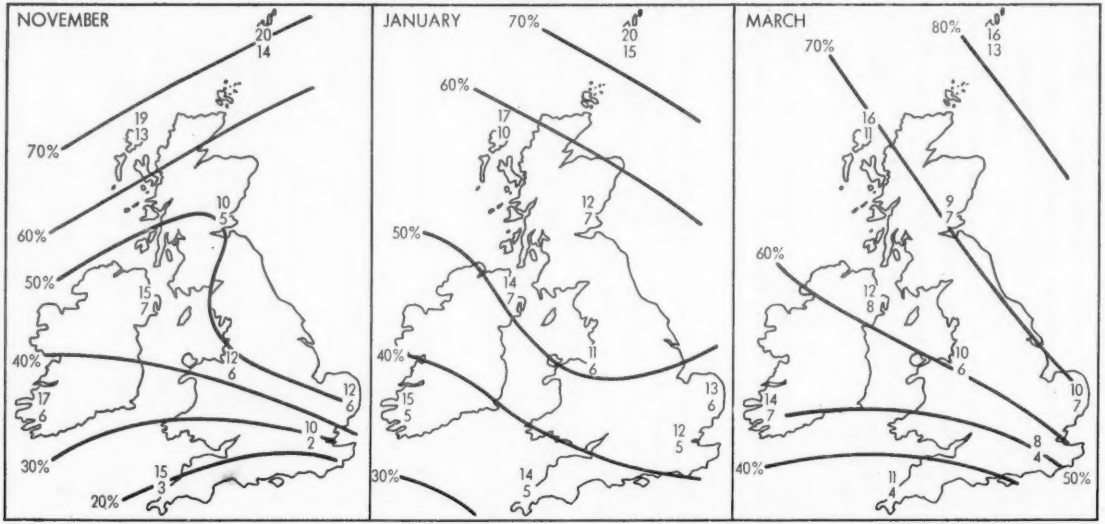


**FIGURE 7—10-YEAR AVERAGE PERCENTAGES OF OCCASIONS ON WHICH THE MELTING LEVEL INTERSECTS THE RADAR BEAM, ELEVATED AT 0·5°, AT A DISTANCE OF 50 km OR LESS**

The height of the radar site is taken as 100 m above sea level. The 12-hour rainfall amount near the radar is between 0·05 mm and 5 mm. The upper figure at each radiosonde station is the mean number of occurrences in the month with rainfall in the given range, and the lower figure is the mean number of occurrences in the month with rainfall in the given range, when the 0°C level intersects the radar beam within 50 km of the radar site.

elevation of 0·5°. The mean freezing-level height over North Wales for rainfall rates between 0·05 and 5 mm in 12 hours in November is 1250 metres, in January is 1100 metres, and in March is 1000 metres. On average, rainfall rates in this range occur on 7 days per month (corresponding approximately to a percentage probability of 35 per cent) in winter, and on 4 of those days or less the freezing level will be intersected at a distance of about 41 km (corresponding to a freezing-level height of 1250 metres which occurs with a percentage probability of about 20 per cent for this rainfall-rate range) in November, 30 km in January and 27 km in March. However, for a rainfall-rate range of 5 mm in 12 hours to 10 mm in 12 hours, the mean heights are 1300 metres in November, 1100 metres in January and 950 metres in March. On average, rainfall rates within this range occur on only 1 day per month or less, and on half of this day the freezing level will be intersected at a distance of about 45 km in November, 36 km in January and 30 km in March.

## 4. Conclusion

The variation of the height of the freezing level over the British Isles in winter, shown in Figure 1 for all rainfall rates, reflects the dependence of this parameter on latitude and season. However, there is a relationship between mean freezing-level height and 12-hour rainfall amount.

The data presented in this paper allow mean estimates to be made of the distance from a radar site at which the freezing level intersects the radar beam in winter. The percentage amount of radar data with 12-hour rainfall in a given range affected by melting level errors may also be estimated.

## 5. Acknowledgements

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